SYSTEM OF LEVELS IN EVEN-EVEN NUCLEI*

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It is of interest, in order to stimulate further experimental and theoretical work, to point out the existence of a system of levels in even-even nuclei whose energies are related by an empirically determined law which seems to be applicable throughout the nuclear chart.

If one plots the ratio R of the energy of the first excited 6+ (denoted $^{1}6+$) and 8+ ($^{1}8+$) states to the energy of the first excited 2+ ($^{1}2+$) state as a function of the ratio $R(^{1}4+)$ of the energy of the first excited 4+ ($^{1}4+$) state to the energy of the first excited 2+ ($^{1}2+$) state, one finds that practically all the points lie on two curves, one for the $^{1}6+$ states and one (tentative) for the $^{1}8+$ states; see Fig. 1. Some of the energy ratio [for $R(^{1}4+)$ larger than 2.9] were taken from Scheuer and Aisenberg's paper[1] with the exception of Gd^{158}.[2] All the values used are given in Table I. Question marks in Fig. 1 mean that there is uncertainty in the assignment of the levels considered. We also show in Fig. 1 the obvious lines for $R(^{1}4+)$ and $R(^{1}2+) = 1$ in order to visualize better the position of the $^{1}2+$, $^{1}4+$, $^{1}6+$, $^{1}8+$ levels. Naturally the $^{1}0+$ ground state is at $R = 0$.

As one would expect, the $R(^{1}8+)$ and $R(^{1}6+)$ curves coincide over a range of $R(^{1}4+)$ values with the straight lines $R(^{1}8+) = -312/7 + (594/35)R(^{1}4+)$ and $R(^{1}6+) = -11 + (27/5)R(^{1}4+)$. These are a direct consequence of

$$E_I = aI(I+1) - b[I(I+1)]^2, \qquad (1)$$

which is the expression for the energy of a rotational state in an axially symmetric nucleus[3] taking into account the rotational-vibrational[4] interaction, where a and b are constants and I the spin of the state. The coefficients in the expressions for $R(^{1}4+)$ and $R(^{1}6+)$ are independent of a and b. The curves show that the rotational nuclei are confined to $R(^{1}4+)$ values larger than or equal to 3.27 because for values smaller than this the empirical curves deviate from (1). Of the rotational nuclei, Hf^{178}, U^{232}, U^{236}, and Pu^{240} do not fall on the theoretical curve (1) within the experimental errors.

The theory of Davydov and Filippov[5] which treats rotations of nonaxially-symmetric nuclei, extends the agreement between theoretical and empirical curves down to values of $R(^{1}4+) = 8/3$ because $R(^{1}4+)$ and $R(^{1}6+)$ are reduced from 10/3 and 7, respectively, for $\gamma = 0°$ to 8/3 and ~5.0 for $\gamma = 30°$. Unfortunately the precise predictions for $R(^{1}6+)$ have not been published so that a more careful comparison with experimental values cannot be made. The $R(^{2}2+)$ (second 2+ excited state) values

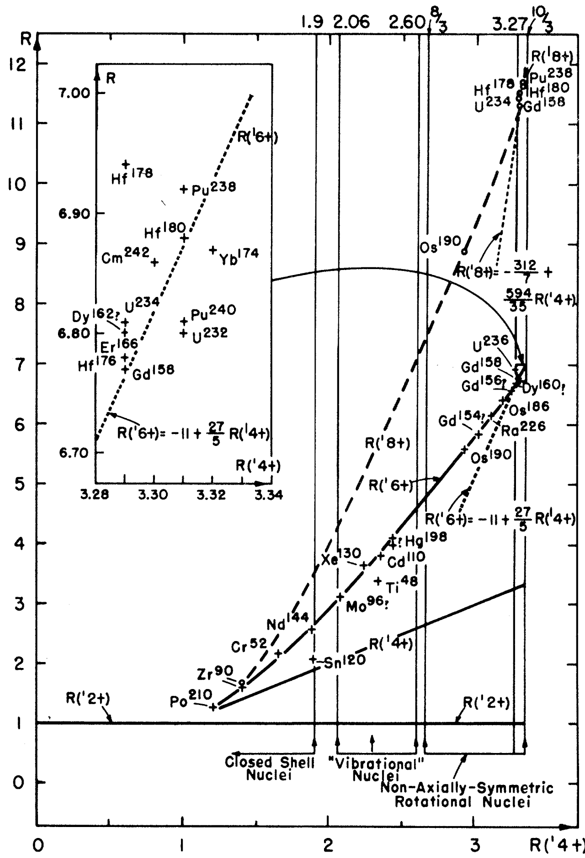

FIG. 1. The figure shows the ratios $R(^{1}8+)$, $R(^{1}6+)$, $R(^{1}4+)$, and $R(^{1}2+)$ as a function of the ratio $R(^{1}4+)$. Experimental points are indicated with a circle for $R(^{1}8+)$ and a plus sign for $R(^{1}6+)$. The $R(^{1}6+)$ points for $R(^{1}4+)$ values in the range from 3.28 to 3.33 are shown in a magnified scale at the upper-left side of the figure. Experimental errors are not indicated in order to avoid confusion in the graph. They are given in Table I. Question marks mean that there is an uncertainty in the assignment of the levels considered. Full lines are experimental; the dashed line is tentative for $R(^{1}8+)$. The dotted lines are the theoretical predictions for rotational states in axially symmetric nuclei taking into account the rotational-vibrational interaction.

Table I. Experimental energy ratios with their errors used in Fig. 1.

	$E(^{1}2+ - ^{1}0+)$ (Mev)	$R(^{1}4+)$	$R(^{1}6+)$	$R(^{1}8+)$
Po^{210}	1.183	1.21 ± 0.01	1.25 ± 0.01	...
Zr^{90}	2.197	1.41 ± 0.03	1.57 ± 0.03	1.64 ± 0.04
Cr^{52}	1.433	1.65 ± 0.01	2.17 ± 0.02	...
Sn^{120}	1.170	1.89 ± 0.02	2.06 ± 0.02	...
Nd^{144}	0.696	1.89 ± 0.01	2.56 ± 0.02	...
Mo^{96}	0.778	2.08 ± 0.03	3.10 ± 0.04	...
Xe^{130}	0.534	2.24 ± 0.02	3.64 ± 0.03	...
Ti^{48}	0.986	2.33 ± 0.02	3.39 ± 0.02	...
Cd^{110}	0.656	2.35 ± 0.02	3.78 ± 0.03	...
Hg^{198}	0.41176	2.43 ± 0.02	3.97 ± 0.03 4.09 ± 0.03 } ?	...
Gd^{154}	0.1229	3.020 ± 0.005	5.83 ± 0.01	...
Gd^{156}	0.08897	3.240 ± 0.005	6.57 ± 0.01	...
Gd^{158}	0.07956	3.292 ± 0.003	6.774 ± 0.006	11.30 ± 0.02
Dy^{160}	0.087	3.27 ± 0.01	6.69 ± 0.01	...
Dy^{162}	0.0808	3.290 ± 0.005	6.80 ± 0.02	...
Er^{166}	0.0805	3.290 ± 0.005	6.80 ± 0.08	...
Yb^{174}	0.0766	3.32 ± 0.02	6.87 ± 0.03	...
Hf^{176}	0.08835	3.290 ± 0.005	6.77 ± 0.13	...
Hf^{178}	0.09319	3.29 ± 0.01	6.94 ± 0.03	11.50 ± 0.09
Hf^{180}	0.09328	3.31 ± 0.01	6.88 ± 0.02	11.64 ± 0.06
Os^{186}	0.13721	3.18 ± 0.06	6.39 ± 0.12	...
Os^{190}	0.187	2.92 ± 0.02	5.59 ± 0.06	8.88 ± 0.08
Ra^{226}	0.06776	3.10 ± 0.01	6.14 ± 0.02	...
U^{232}	0.0472	3.31 ± 0.02	6.80 ± 0.04	...
U^{234}	0.04350	3.29 ± 0.02	6.81 ± 0.03	11.47 ± 0.09
U^{236}	0.04528	3.26 ± 0.03	6.92 ± 0.12	...
Pu^{238}	0.04407	3.310 ± 0.005	6.92 ± 0.02	11.68 ± 0.23
Pu^{240}	0.04287	3.310 ± 0.005	6.810 ± 0.005	...
Cm^{242}	0.04212	3.30 ± 0.01	6.86 ± 0.12	...

of Davydov and Filippov as a function of $R(^{1}4+)$ values seem to be in agreement with experimental results. The agreement of the theoretical predictions for transition probability ratios and $E2$, $M1$ admixtures in the $^{2}2+ \rightarrow ^{1}2+$ transition is quite good.[6,7] It is interesting to note that the smallest $R(^{1}4+)$ predicted by this theory is 8/3; clearly many nuclei have $R(^{1}4+)$ smaller than 8/3 (see Fig. 1) so that they are not in agreement with this theory.

The "vibrational" nuclei[8] have average $R(^{1}4+)$ value of 2.3[9] with values ranging from 2.07 to 2.63 and the closed shell nuclei have $R(^{1}4+)$ values smaller than 1.9.[9] The nuclei above the "vibrational" nuclei have been called nonaxially-symmetric rotational nuclei in Fig. 1. Of all the nuclei which are not nonaxially-symmetric rotational nuclei only Ti^{48} and Sn^{120} do not fall within their experimental error on the empirical curve.

The classification of nuclei into different types seems not to be essential, at least with respect to this system of levels, because all nuclei follow the same curves which relate the energies of these excited states.

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ANTIPARTICLES IN SPACE

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About a decade ago, Hoyle[1] obtained a steady-state model for an expanding universe by introducing continuous creation of matter into the field equations of general relativity. More recently, Morrison[2] has speculated that the creation of matter throughout the universe may take place by formation of particle, antiparticle pairs which subsequently are segregated by a repulsive force between matter and antimatter. Burbidge and Hoyle[3] have estimated that several observed astrophysical phenomena (both within our galaxy and outside it) might be explained as due to the presence of antimatter. But they conclude that the ratio of antimatter to ordinary matter in our galaxy cannot exceed $\sim 10^{-7}$. The purposes of this Letter are (a) to point out that a slightly more stringent limit for this ratio is suggested by cosmic-ray measurements reported by Perlow and Kissinger[4,5] and (b) to observe that relatively simple equipment in earth satellites or in space probes should serve either to detect antiparticles in our galaxy or to define a much more stringent limit on their number density than can presently be set.

If one assumes that antiprotons exist throughout the galaxy in the limiting number density, n_-, of $10^{-7}/\mathrm{cm}^3$ estimated by Burbidge and Hoyle and if one takes the $p, \bar{p}$ annihilation cross section used by them and adopts, for protons in the galaxy, the number density, n_+, of 1 per cm^3, one obtains a $p, \bar{p}$ annihilation rate of 3×10^{-22} $\mathrm{cm}^{-3}\ \mathrm{sec}^{-1}$. From an average annihilation, there ultimately appear 3.6 photons of ~ 170 Mev and 3.6 photons of 0.51 Mev.[6] Therefore, the source of gamma rays of each type has an intensity, throughout the galaxy, of $10.8\times10^{-22}\ \mathrm{cm}^{-3}\ \mathrm{sec}^{-1}$. The flux, Q ($\mathrm{cm}^{-2}\ \mathrm{sec}^{-1}$) of these photons coming from a spherical region of radius R (cm) surrounding the earth is

$$Q = \tfrac{1}{2}PR\ \mathrm{cm}^{-2}\ \mathrm{sec}^{-1}, \qquad (1)$$

where P is the photon generation rate in the source ($\mathrm{cm}^{-3}\ \mathrm{sec}^{-1}$). We can obtain, simply, a very conservative value for the flux, by calculating those coming from a sphere of 10 000 light years (10^{22} cm) radius. (The flux from the whole galaxy would certainly be several times larger than this.) We find, for each type of photon (170 Mev and 0.51 Mev),

$$Q = \tfrac{1}{2}(10.8\times10^{-22}\times10^{22}) = 5.4\ \mathrm{photons/cm^2\ sec}.$$

In terms of energy flux, this is $\sim 900\ \mathrm{Mev/cm^2\ sec}$ for the 170-Mev photons and $\sim 2.5\ \mathrm{Mev/cm^2\ sec}$ for the 0.51-Mev photons.

Perlow and Kissinger measured the primary gamma radiation at 80 km altitude and found the flux of photons in the energy range 0.1 to 15 Mev to be $\leqq 2.7/\mathrm{cm^2\ sec}$ from the upper hemisphere.[4] This figure seems to be in fair agreement with the calculated flux of 0.51-Mev photons until one recalls that (a) the calculated flux is not that from the whole galaxy but only from a 10^{22}-cm sphere and (b) the measurements included not only 0.51-Mev photons but all photons between 0.1 and 15 Mev. From these considerations one is probably justified in saying that $n_-/n_+ < 10^{-8}$ in the galaxy. Perlow and Kissinger measured, also, the flux of gamma rays of energy between 3.4 and 90 Mev and found an energy flux of 1.4 $\mathrm{Mev/cm^2\ sec}$ from the upper hemisphere.[5] It is not possible to make a very meaningful comparison of these data with the calculated flux of 170-Mev gamma rays because the latter (if they occur) would not have been detected very efficiently by the apparatus used. However, the fact that evidence of this large energy flux (~ 900 $\mathrm{Mev/cm^2\ sec}$) has not been found in other cosmic-ray experiments indicates that the flux of ~ 170-Mev gammas is at least one or two orders of magnitude smaller than that calculated above ($5.4/\mathrm{cm^2\ sec}$).